

Bernd Aulbach—Obituary

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Professor Bernd Aulbach

1. Biography

1.1 Family and Education

On January 14, 2005, Professor Bernd Aulbach suddenly and unexpectedly passed away at the age of 57 years. The mathematical community lost a respected scientist, a valued colleague, and a popular university teacher. Bernd Aulbach was born in Aschaffenburg on December 23, 1947, where he also went to school and graduated with the German high school diploma (Abitur) in 1967. Subsequently, he studied mathematics with a minor in physics in Würzburg. In the year 1970, still as a student, he married Gudrun Nöll with whom he had one daughter and two sons; the children were born in the years 1971, 1976 and 1980.

1.2 Scientific career

Bernd Aulbach discovered his interest in differential equations “early” and consequently wrote his Master’s thesis under Professor Hans Wilhelm Knobloch on the topic: “The Domain of Attractivity of an Asymptotically Stable Solution for Non-autonomous Periodic Differential Equations”. He graduated from Würzburg with the Master’s degree (Diploma) in 1973 and remained in Würzburg as a scientific assistant to Professor Knobloch, with whom

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he completed the PhD degree in 1976 by writing a thesis, also on domains of attractivity of stable periodic solutions. He spent the academic year 1978/1979 as a Visiting Assistant Professor at State University of New York in Albany. From 1983 until 1986, he had a fellowship from the foundation “Volkswagenwerk Stiftung” to work on the project “Qualitative Analysis of Nonlinear Dynamic Systems by Means of Invariant Manifolds”. In the context of this project, his Habilitationsschrift “Continuous and Discrete Dynamics near Manifolds of Equilibria” emerged, which also appeared as lecture notes published by Springer in 1984. In August 1984, he became a lecturer (Privatdozent) at the University of Würzburg. Subsequently, he was awarded a highly competitive Heisenberg scholarship from Deutsche Forschungsgemeinschaft (DFG, German Research Society), which he used to finance a longer stay at the University of California in Berkeley in 1986/1987. In the year 1987, he finally accepted a position at the University of Augsburg, which he held until his death.

1.3 Professor in Augsburg

1.3.1 Academic teacher. Professor Aulbach was a highly gifted teacher and consequently was very popular among his students. This also resulted in a higher-than-average number of graduate students. As a colleague, Bernd Aulbach was extremely cooperative, and he never refused to serve in committees and advisory commissions. His balanced personality was esteemed, and his ideas advanced many a committee.

A fruitful reward of his careful lecturing is his textbook *Gewöhnliche Differenzialgleichungen* on ordinary differential equations which was very well received and is still much in demand; the second, revised and amended edition has just appeared. This introductory textbook develops the theory systematically for non-autonomous equations. His didactical strength including elucidating graphical illustrations is fully apparent. In the proofs everything is clearly spelled out, nothing is left in the dark or obscure.

His students also have profited from his lecture notes on the first-year course in analysis [73], where starting from the basics on logic, sets, etc. he immediately introduced the appropriate framework for modern analysis. Convergence and continuity are treated in metric spaces, and differentiation and integration are treated in \mathbb{R}^N (instead of following the traditional German path of devoting the first semester to analysis on the real line). He resisted the temptation to include many sidetracks; thus the text is very suitable as the basis for a course (and a number of colleagues have used it). A hint to his interest in mathematics teachers education is his careful (300 pp.) preparation of solutions to all state exams for high school teachers given in Bavaria between 1989 and 1999 which he compiled with his student Stefan Keller.

He had a talent to attract gifted students. He meticulously planned sequences of courses which allowed him to bring students to the forefront of research. In spite of his very open and easy manners, he always was very conscious of his role as an academic teacher.

1.3.2 Graduiertenkolleg (Research training group). The Graduate school in “Nonlinear Problems in Analysis, Geometry, and Physics” was awarded to the University of Augsburg in 1996 by DFG, and from the very beginning, Bernd Aulbach was its speaker. Together with Fritz Colonius he also directed the working group “Dynamics and Control of Ordinary Differential Equations”. As a speaker he mastered the difficult task of bringing together different scientific agenda and a number of very different personalities into a common

context, for the benefit of the next generation of scientists. In particular, it was his merit to install the weekly Graduiertenkolloquium as the central forum for scientific exchange. It was also mainly due to his dedicated input that the Graduate School was extended twice and thus the maximum support duration was granted. Unfortunately, he was unable to see his favorite project (as he admitted) through to its completion in the year 2005.

1.4 International society of difference equations (ISDE)

Bernd Aulbach played a leading role in the promotion and development of the area of difference equations. Since the convening of the first international conference on difference equations and applications (ICDEA1) in San Antonio, 1994, Bernd Aulbach has been involved in all subsequent ICDEA meetings. And in 2001, he organized ICDEA6 in Augsburg which attracted the leading mathematicians in the area and is considered the benchmark for all future meetings to emulate. In this meeting, Aulbach founded the international society of difference equations (ISDE) and was elected its first president. Two years later at ICDEA8 in Brno, he was reelected as the president of the society and unfortunately he did not complete his two-year term. Due to Aulbach's high stature in the mathematical community, ISDE grew rapidly and gained wide respectability in the mathematical community.

Finally, Aulbach was one of the organizers of the international conference on difference equations, special functions, and applications which was later held at Munich in 2005 after his death.

1.5 The Journal of Difference Equations and Applications (JDEA)

Bernd Aulbach was one of the founders of the *Journal of Difference Equations* and served as its consulting editor since its inception in 1995. He has been involved extensively in the editorial activities of the journal, which contributed significantly to the high reputation of the journal. On his last day in the office, he had ten papers from JDEA in various stages of refereeing.

2. Scientific contributions

2.1 Zubov's method

Bernd Aulbach began his mathematical career in stability theory of differential equations. Once, a Lyapunov function is known, it is a simple task to decide if a given point was attracted to an equilibrium. However, the construction of Lyapunov functions are notoriously difficult, in particular, when a Lyapunov function is sought on the whole (asymptotic) stability region. In the real analytic context, Zubov had presented a general construction method allowing to determine approximately this complete asymptotic stability region. It is given by the maximal region of analyticity of an optimal Lyapunov function obtained as the solution of a Cauchy problem for a first order PDE. Then, using partial sums of series expansions, a sequence of Lyapunov functions was obtained providing a sequence of regions approximating the region of asymptotic stability from inside. Aulbach's dissertation, under the direction of Hans Wilhelm Knobloch, and later extensions in [60,61,69] developed analogous results for equilibria of differential equations with almost periodic time dependence and for orbitally asymptotically stable periodic solutions of autonomous

differential equations. The treatment of the latter case also required properties of integral manifolds obtained in reference [68].

As a side remark note that in a less smooth context, constructive Lyapunov methods and, in particular, numerical versions of Zubov's method have recently found much interest in control theory; although Aulbach did not work in this field, he always was well aware of the close relations to this area and the relevance of his work for control theorists.

2.2 *Manifolds of equilibria, asymptotic phases, and biology*

The general problem of asymptotic phase for a compact invariant set is to decide whether, for each solution $x(t)$ approaching M as $t \rightarrow \infty$, there exists a solution $\hat{x}(t)$ in M such that $|x(t) - \hat{x}(t)| \rightarrow 0$ as $t \rightarrow \infty$. The paper [65] considered the case of autonomous differential equations under hyperbolicity assumptions together with an assumption on the behavior within M . This topic is also taken up in a later paper with Flockerzi and Knobloch [49].

The Springer lecture in mathematics [75] (essentially the habilitation thesis at the University of Würzburg) presents a theory of dynamics near manifolds of equilibria. Both non-autonomous differential equations and (autonomous) difference equations are treated; here also non-invertible maps are allowed thus extending results of Hirsch, Pugh and Shub on diffeomorphisms. The main question is: given a manifold of equilibria which is approached by a trajectory, when can one guarantee that just one of the equilibria is picked as the ω -limit set? The starting point is "a prototype result" providing three sufficient conditions which later are "generalized, modified and applied" to problems in population genetics. These are the basic selection problem considered earlier in a joint paper with Hadelar [59] and the discrete time-Fisher–Wright–Haldane model [58].

Although, Lyapunov had already discussed the case of a curve of asymptotically stable equilibria, the general case was open until Aulbach's work. The book [75] refrains from treating the most general case of invariant sets, but notes that many problems can be reformulated as non-autonomous problems with manifolds of equilibria. This is a continuing characteristic of Aulbach's work: instead of working with an abstract machinery he preferred to distill the essence of an argument or a concept. Often this also resulted in striking paper titles (cf. [11,56] or [20]). The timeliness of this topic can also be seen from the fact that, independently, similar results on asymptotic phases were proved by Shoshitaishvili.

Later, Aulbach came back to biological problems in a joint paper with Flockerzi on the dynamics of Eigen's hypercycles [45]. Also in other respects, the book touches upon the themes which will later dominate Aulbach's work: the parallel treatment of continuous and discrete dynamics, non-autonomous dynamics, and invariant manifolds. Thus, the stage is set for many things to come in the following twenty years.

2.3 *Smoothness of center manifolds*

The smoothness problem was addressed again and again in Aulbach's scientific work. To what extent does an invariant manifold or fiber bundle inherit a certain degree of smoothness from the data of the underlying dynamical system? The example of a center manifold that inherits C^k -smoothness but not C^∞ -smoothness illustrates the intricacy of the problem. In the 1980s, Aulbach mainly turned his interest towards the question of analyticity of center manifolds [50,54]. His lasting interest in this area of research was reflected by his and his students further contributions [10,40].

2.4 Time scales

A recurring topic in Bernd Aulbach's work of the early 1980s was the parallelism in discrete and continuous dynamical systems (See "Habilitationsschrift" [75]). Also in his two Würzburg courses on continuous (1985) and discrete dynamics (1986) Aulbach often pointed out numerous analogies and surprising discrepancies arising in the two different settings. So, he strongly supported the idea of his first PhD student Hilger that emerged in 1987, to develop an abstract and general calculus embracing both time scales, \mathbb{R} and \mathbb{Z} . The first articles on the resulting so-called "Calculus on Measure Chains", including [42,43], did not find great attention in the early 1990s. Aulbach and Hilger had to give up any further reaching research projects. It was only in the late 1990s that a worldwide interest in time scales calculus—this alternative notion was proposed by Aulbach—arose. Until 2005 three monographs, several hundred articles and a cover story in "The New Scientist" (179/2003) appeared. Calculus on measure chains/time scales also served as a framework for several diploma theses supervised by Aulbach, [5], and two PhD theses of Pötzsche and Keller [14].

2.5 Linearization and discretization

Papers [34,36,39,41] coauthored by Aulbach and Garay, deal with systems in Banach spaces with a dominant linear term and Hartman-Grobman like robustness results. In [34] one-step discretizations of semilinear non-autonomous differential equations for which the linear part has an exponential dichotomy are investigated, explicit estimates in terms of stepsize and order of the discretization method are obtained and the connection with shadowing results are discussed. The other papers involve maps of a Banach space into itself with a fixed point without assuming that the derivative at the fixed point is invertible. Non-invertibility as a major obstacle and interesting phenomenon in qualitative analysis of discrete dynamical systems was tackled later again by Aulbach, also together with his PhD student Jürgen Kalkbrenner [11,16]. In [36,39] partial linearization theorems concerning the strongly expanding or contracting components of the maps are obtained using elementary degree theoretic arguments, for example. This also provides a new proof of the Hartman–Grobman theorem in the finite dimensional case. In [41], partial linearization and partial decoupling results are proved and a counterexample is given to show that full linearization and decoupling is not always possible.

2.6 Evolutionary semi-groups and almost periodicity

Papers [13,18,22–24,27,29,31] are coauthored by Aulbach and Minh, while papers [30,35,37,38] are coauthored by Aulbach, Minh and Zabreiko. The first group of papers involves Banach space valued differential equations, which have almost periodic time dependent coefficients. The existence of solutions, in particular of almost periodic solutions is of major interest as well as the generation of evolutionary semi-groups. The exponential stability and robustness of solutions to perturbations is also considered as well as the spectrum of the linear part of the equations assuming hyperbolicity or the existence of an exponential dichotomy. Several papers focus attention on delay and functional differential equations.

Paper [24] establishes the structural stability of non-autonomous linear difference equations in a Hilbert space with an invertible operator assuming an exponential dichotomy.

Papers [30,31] investigate the hyperbolicity and spectrum of the translation operator along solutions of linear systems.

2.7 Integral manifolds, exponential dichotomies and dichotomy spectrum

A principal topic in Aulbach's work are invariant manifolds or integral manifolds as they are also called in the context of time-varying or non-autonomous differential equations, since in the non-autonomous case they consist of integral or solution curves. Together with his former PhD adviser Knobloch he elaborated the role of invariant manifolds for singularly perturbed systems [55].

Invariant manifolds consist of solutions with prescribed exponential growth rates and form the dynamical skeleton of an underlying system. They are a well established tool of qualitative dynamical systems theory dating back to Hadamard and Perron who introduced the graph transformation and Lyapunov–Perron method in the beginning of the 20th century. The overall presence of invariant manifolds in dynamical systems caused Anosov to write in 1969: “Every five years or so, if not more often, someone discovers’ the theorem of Hadamard and Perron, proving it either by Hadamard’s method of proof or by Perron’s”.

It was one of Bernd Aulbach's contributions to develop and present the theory in a way to make it very attractive and accessible to students as well as extending the theory to various classes of non-autonomous equations, e.g. Carathéodory type equations with only measurable time dependence [26] or discrete time difference equations [28]. This also made generalizations to random dynamical systems possible (see the thesis by Wanner (1993)).

Stable, center and unstable manifolds of a rest point are typical examples of invariant manifolds. They are locally approximated by corresponding subspaces of the linearization at the rest point. In the autonomous case the subspace which is tangential to the stable manifold is the sum of the generalized eigenspaces to all eigenvalues with negative real part. In the non-autonomous case eigenvalues and eigenvectors are useless in general, instead uniform exponential estimates are used to generalize stable, center and unstable subspaces for linear systems which then leads to a robust notion [51]. Aulbach emphasized that in the presence of a central subspace (generalizing eigenvalues on the imaginary axis) there is “Trouble with linearization” [56]—local behavior depends on nonlinear terms—and he also investigated the case of eigenvalues close to the imaginary axis [44].

Under the influence of the seminal work of Coppel, Aulbach fully realized that exponential dichotomies are the right notion generalizing hyperbolicity to the non-autonomous setting. The analysis of exponential dichotomies and dichotomy spectrum was carried out in [12,15] and on the level of evolutionary semigroups in [27,35]. In his papers about invariant manifolds, Aulbach assumed that the linearization is decoupled and satisfies an exponential dichotomy. Under these assumptions, he proved together with Wanner for differential equations of Carathéodory type, i.e. equations with only measurable time dependence, the existence of invariant manifolds [26] and foliations [7] which then can be used to prove a non-autonomous version of the Hartman–Grobman theorem in Banach spaces [19,25]. These results are generalized to the discrete time case with not necessarily invertible right-hand side in [21,8] by constructing so-called fiber bundles. The smoothness of these fiber bundles was established in [10]. Precursors of Aulbach's results on invariant manifolds and fiber bundles can already be found in an early paper with Flockerzi [47]. Besides stable, unstable and center manifolds, Aulbach stressed the existence of strongly stable, strongly

unstable and even more general invariant manifolds leading to a hierarchy of invariant manifolds [48] and fiber bundles [33].

Lately, he studied with Rasmussen and Siegmund the relation of invariant manifolds to pullback attractors of non-autonomous differential [1,2] and difference [3] equations.

2.8 Discrete dynamics—role of invertibility, chaos

Aulbach's overall knowledge enabled him to stimulate research in areas of difference equations theory, that were not in the main focus of his interests. He coauthored papers of Kalkbrenner and Kieninger on aspects of chaos theory [6,17,32]. Recently Aulbach, Elaydi and Ziegler [4] showed how advanced methods of difference equations theory can be exploited in mathematical physics. They analyzed the asymptotic behavior of an eigenfunction recursion equation of Schrödinger type modeling the motion of a random mass particle.

Bernd Aulbach had still many plans to follow which were made impossible by his sudden and premature death. This is apparent in a number of joint papers, which appear posthumously. Furthermore, in the last year, he had given courses on complex function theory and on dynamics in the complex plane with the aim to write a book, which was to bridge the gap between the standard complex function theory courses and the advanced research literature. Here and in many other respects, the mathematical community will miss his insight, his didactical skills and his personality.

3. List of doctoral students

- Martin Rasmussen: Attractivity and Bifurcation for Nonautonomous Dynamical Systems (Augsburg 2005, in preparation).
- Bernd Kieninger: Iterated Function Systems on Compact Hausdorff Spaces (Augsburg 2002).
- Christian Pötzsche: Langsame Mannigfaltigkeiten dynamischer Gleichungen auf Maßketten (Augsburg 2002).
- Stefan Keller: Asymptotisches Verhalten invarianter Faserbündel bei Diskretisierung und Mittelwertbildung im Rahmen der Analysis auf Zeitskalen (Augsburg 1999).
- Stefan Siegmund: Spektral-Theorie, glatte Faserungen und Normalformen für Differentialgleichungen vom Caratheodory-Typ (Augsburg 1999).
- Jürgen Kalkbrenner: Exponentielle Dichotomie und chaotische Dynamik nichtinvertierbarer Differenzengleichungen (Augsburg 1994).
- Thomas Wanner: Linearization of Random Dynamical Systems (Augsburg 1993).
- Stefan Hilger: Ein Masskettenkalkül mit Anwendungen auf Zentrumsmannigfaltigkeiten (Würzburg 1989).

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